1.A palindromic number reads the same both ways. The largest palindrome made from the product of two 2-digit numbers is 9009 = 91 × 99.

Find the largest palindrome made from the product of two 3-digit numbers.

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2.The prime factors of 13195 are 5, 7, 13 and 29. Find the largest palindrome made from the product of two 3-digit numbers.

A palindromic number reads the same both ways. The largest palindrome made from the product of two 2-digit numbers is 9009 = 91 × 99.

What is the largest prime factor of the number 60085

3. 2520 is the smallest number that can be divided by each of the numbers from 1 to 10 without any remainder.

What is the smallest positive number that is *evenly divisible* by all of the numbers from 1 to 20?

4. The sum of the squares of the first ten natural numbers is,

12 + 22 + ... + 102 = 385

The square of the sum of the first ten natural numbers is,

(1 + 2 + ... + 10)2 = 552 = 3025

Hence the difference between the sum of the squares of the first ten natural numbers and the square of the sum is 3025 − 385 = 2640.

Find the difference between the sum of the squares of the first one hundred natural numbers and the square of the sum.

5. By listing the first six prime numbers: 2, 3, 5, 7, 11, and 13, we can see that the 6th prime is 13.

What is the 10 001st prime number?

6. The four adjacent digits in the 1000-digit number that have the greatest product are 9 × 9 × 8 × 9 = 5832.

73167176531330624919225119674426574742355349194934  
96983520312774506326239578318016984801869478851843  
85861560789112949495459501737958331952853208805511  
12540698747158523863050715693290963295227443043557  
66896648950445244523161731856403098711121722383113  
62229893423380308135336276614282806444486645238749  
30358907296290491560440772390713810515859307960866  
70172427121883998797908792274921901699720888093776  
65727333001053367881220235421809751254540594752243  
52584907711670556013604839586446706324415722155397  
53697817977846174064955149290862569321978468622482  
83972241375657056057490261407972968652414535100474  
82166370484403199890008895243450658541227588666881  
16427171479924442928230863465674813919123162824586  
17866458359124566529476545682848912883142607690042  
24219022671055626321111109370544217506941658960408  
07198403850962455444362981230987879927244284909188  
84580156166097919133875499200524063689912560717606  
05886116467109405077541002256983155200055935729725  
71636269561882670428252483600823257530420752963450

Find the thirteen adjacent digits in the 1000-digit number that have the greatest product. What is the value of this product?

7. A Pythagorean triplet is a set of three natural numbers, *a* < *b* < *c*, for which,

*a*2 + *b*2 = *c*2

For example, 32 + 42 = 9 + 16 = 25 = 52.

There exists exactly one Pythagorean triplet for which *a* + *b* + *c* = 1000.  
Find the product *abc*.

8. The sum of the primes below 10 is 2 + 3 + 5 + 7 = 17.

Find the sum of all the primes below two million.

9. In the 20×20 grid below, four numbers along a diagonal line have been marked in red.

08 02 22 97 38 15 00 40 00 75 04 05 07 78 52 12 50 77 91 08  
49 49 99 40 17 81 18 57 60 87 17 40 98 43 69 48 04 56 62 00  
81 49 31 73 55 79 14 29 93 71 40 67 53 88 30 03 49 13 36 65  
52 70 95 23 04 60 11 42 69 24 68 56 01 32 56 71 37 02 36 91  
22 31 16 71 51 67 63 89 41 92 36 54 22 40 40 28 66 33 13 80  
24 47 32 60 99 03 45 02 44 75 33 53 78 36 84 20 35 17 12 50  
32 98 81 28 64 23 67 10 **26** 38 40 67 59 54 70 66 18 38 64 70  
67 26 20 68 02 62 12 20 95 **63** 94 39 63 08 40 91 66 49 94 21  
24 55 58 05 66 73 99 26 97 17 **78** 78 96 83 14 88 34 89 63 72  
21 36 23 09 75 00 76 44 20 45 35 **14** 00 61 33 97 34 31 33 95  
78 17 53 28 22 75 31 67 15 94 03 80 04 62 16 14 09 53 56 92  
16 39 05 42 96 35 31 47 55 58 88 24 00 17 54 24 36 29 85 57  
86 56 00 48 35 71 89 07 05 44 44 37 44 60 21 58 51 54 17 58  
19 80 81 68 05 94 47 69 28 73 92 13 86 52 17 77 04 89 55 40  
04 52 08 83 97 35 99 16 07 97 57 32 16 26 26 79 33 27 98 66  
88 36 68 87 57 62 20 72 03 46 33 67 46 55 12 32 63 93 53 69  
04 42 16 73 38 25 39 11 24 94 72 18 08 46 29 32 40 62 76 36  
20 69 36 41 72 30 23 88 34 62 99 69 82 67 59 85 74 04 36 16  
20 73 35 29 78 31 90 01 74 31 49 71 48 86 81 16 23 57 05 54  
01 70 54 71 83 51 54 69 16 92 33 48 61 43 52 01 89 19 67 48

The product of these numbers is 26 × 63 × 78 × 14 = 1788696.

What is the greatest product of four adjacent numbers in the same direction (up, down, left, right, or diagonally) in the 20×20 grid?

10. The sequence of triangle numbers is generated by adding the natural numbers. So the 7th triangle number would be 1 + 2 + 3 + 4 + 5 + 6 + 7 = 28. The first ten terms would be:

1, 3, 6, 10, 15, 21, 28, 36, 45, 55, ...

Let us list the factors of the first seven triangle numbers:

**1**: 1  
**3**: 1,3  
**6**: 1,2,3,6  
**10**: 1,2,5,10  
**15**: 1,3,5,15  
**21**: 1,3,7,21  
**28**: 1,2,4,7,14,28

We can see that 28 is the first triangle number to have over five divisors.

What is the value of the first triangle number to have over five hundred divisors?

11. he following iterative sequence is defined for the set of positive integers:

*n* → *n*/2 (*n* is even)  
*n* → 3*n* + 1 (*n* is odd)

Using the rule above and starting with 13, we generate the following sequence:

13 → 40 → 20 → 10 → 5 → 16 → 8 → 4 → 2 → 1

It can be seen that this sequence (starting at 13 and finishing at 1) contains 10 terms. Although it has not been proved yet (Collatz Problem), it is thought that all starting numbers finish at 1.

Which starting number, under one million, produces the longest chain?

**NOTE:** Once the chain starts the terms are allowed to go above one million.

12. Published on Friday, 3rd May 2002, 06:00 pm; Solved by 129077; Difficulty rating: 5%

13. 215 = 32768 and the sum of its digits is 3 + 2 + 7 + 6 + 8 = 26.

What is the sum of the digits of the number 21000?

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Bottom of Form

14. If the numbers 1 to 5 are written out in words: one, two, three, four, five, then there are 3 + 3 + 5 + 4 + 4 = 19 letters used in total.

If all the numbers from 1 to 1000 (one thousand) inclusive were written out in words, how many letters would be used?

**NOTE:** Do not count spaces or hyphens. For example, 342 (three hundred and forty-two) contains 23 letters and 115 (one hundred and fifteen) contains 20 letters. The use of "and" when writing out numbers is in compliance with British usage.

15. y starting at the top of the triangle below and moving to adjacent numbers on the row below, the maximum total from top to bottom is 23.

**3**  
**7** 4  
2 **4** 6  
8 5 **9** 3

That is, 3 + 7 + 4 + 9 = 23.

Find the maximum total from top to bottom of the triangle below:

75  
95 64  
17 47 82  
18 35 87 10  
20 04 82 47 65  
19 01 23 75 03 34  
88 02 77 73 07 63 67  
99 65 04 28 06 16 70 92  
41 41 26 56 83 40 80 70 33  
41 48 72 33 47 32 37 16 94 29  
53 71 44 65 25 43 91 52 97 51 14  
70 11 33 28 77 73 17 78 39 68 17 57  
91 71 52 38 17 14 91 43 58 50 27 29 48  
63 66 04 68 89 53 67 30 73 16 69 87 40 31  
04 62 98 27 23 09 70 98 73 93 38 53 60 04 23

**NOTE:** As there are only 16384 routes, it is possible to solve this problem by trying every route. However, [Problem 67](https://projecteuler.net/problem=67), is the same challenge with a triangle containing one-hundred rows; it cannot be solved by brute force, and requires a clever method! ;o)

16. You are given the following information, but you may prefer to do some research for yourself.

* 1 Jan 1900 was a Monday.
* Thirty days has September,  
  April, June and November.  
  All the rest have thirty-one,  
  Saving February alone,  
  Which has twenty-eight, rain or shine.  
  And on leap years, twenty-nine.
* A leap year occurs on any year evenly divisible by 4, but not on a century unless it is divisible by 400.

How many Sundays fell on the first of the month during the twentieth century (1 Jan 1901 to 31 Dec 2000)?

17. *n*! means *n* × (*n* − 1) × ... × 3 × 2 × 1

For example, 10! = 10 × 9 × ... × 3 × 2 × 1 = 3628800,  
and the sum of the digits in the number 10! is 3 + 6 + 2 + 8 + 8 + 0 + 0 = 27.

Find the sum of the digits in the number 100!

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Bottom of Form

18. Let d(*n*) be defined as the sum of proper divisors of *n* (numbers less than *n* which divide evenly into *n*).  
If d(*a*) = *b* and d(*b*) = *a*, where *a* ≠ *b*, then *a* and *b* are an amicable pair and each of *a* and *b* are called amicable numbers.

For example, the proper divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110; therefore d(220) = 284. The proper divisors of 284 are 1, 2, 4, 71 and 142; so d(284) = 220.

Evaluate the sum of all the amicable numbers under 10000.

19. Using [names.txt](https://projecteuler.net/project/resources/p022_names.txt) (right click and 'Save Link/Target As...'), a 46K text file containing over five-thousand first names, begin by sorting it into alphabetical order. Then working out the alphabetical value for each name, multiply this value by its alphabetical position in the list to obtain a name score.

For example, when the list is sorted into alphabetical order, COLIN, which is worth 3 + 15 + 12 + 9 + 14 = 53, is the 938th name in the list. So, COLIN would obtain a score of 938 × 53 = 49714.

What is the total of all the name scores in the file?

20. A perfect number is a number for which the sum of its proper divisors is exactly equal to the number. For example, the sum of the proper divisors of 28 would be 1 + 2 + 4 + 7 + 14 = 28, which means that 28 is a perfect number.

A number *n* is called deficient if the sum of its proper divisors is less than *n* and it is called abundant if this sum exceeds *n*.

As 12 is the smallest abundant number, 1 + 2 + 3 + 4 + 6 = 16, the smallest number that can be written as the sum of two abundant numbers is 24. By mathematical analysis, it can be shown that all integers greater than 28123 can be written as the sum of two abundant numbers. However, this upper limit cannot be reduced any further by analysis even though it is known that the greatest number that cannot be expressed as the sum of two abundant numbers is less than this limit.

Find the sum of all the positive integers which cannot be written as the sum of two abundant numbers.

21. permutation is an ordered arrangement of objects. For example, 3124 is one possible permutation of the digits 1, 2, 3 and 4. If all of the permutations are listed numerically or alphabetically, we call it lexicographic order. The lexicographic permutations of 0, 1 and 2 are:

012   021   102   120   201   210

What is the millionth lexicographic permutation of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9?

22. The Fibonacci sequence is defined by the recurrence relation:

F*n* = F*n*−1 + F*n*−2, where F1 = 1 and F2 = 1.

Hence the first 12 terms will be:

F1 = 1  
F2 = 1  
F3 = 2  
F4 = 3  
F5 = 5  
F6 = 8  
F7 = 13  
F8 = 21  
F9 = 34  
F10 = 55  
F11 = 89  
F12 = 144

The 12th term, F12, is the first term to contain three digits.

What is the first term in the Fibonacci sequence to contain 1000 digits?

23. A unit fraction contains 1 in the numerator. The decimal representation of the unit fractions with denominators 2 to 10 are given:

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| 1/2 | = | 0.5 |
| 1/3 | = | 0.(3) |
| 1/4 | = | 0.25 |
| 1/5 | = | 0.2 |
| 1/6 | = | 0.1(6) |
| 1/7 | = | 0.(142857) |
| 1/8 | = | 0.125 |
| 1/9 | = | 0.(1) |
| 1/10 | = | 0.1 |

Where 0.1(6) means 0.166666..., and has a 1-digit recurring cycle. It can be seen that 1/7 has a 6-digit recurring cycle.

Find the value of *d* < 1000 for which 1/*d* contains the longest recurring cycle in its decimal fraction part.

24. Euler discovered the remarkable quadratic formula:

*n*² + *n* + 41

It turns out that the formula will produce 40 primes for the consecutive values *n* = 0 to 39. However, when *n* = 40, 402 + 40 + 41 = 40(40 + 1) + 41 is divisible by 41, and certainly when *n* = 41, 41² + 41 + 41 is clearly divisible by 41.

The incredible formula  *n*² − 79*n* + 1601 was discovered, which produces 80 primes for the consecutive values *n* = 0 to 79. The product of the coefficients, −79 and 1601, is −126479.

Considering quadratics of the form:

*n*² + *an* + *b*, where |*a*| < 1000 and |*b*| < 1000

where |*n*| is the modulus/absolute value of *n*  
e.g. |11| = 11 and |−4| = 4

Find the product of the coefficients, *a* and *b*, for the quadratic expression that produces the maximum number of primes for consecutive values of *n*, starting with *n* = 0.

25. Starting with the number 1 and moving to the right in a clockwise direction a 5 by 5 spiral is formed as follows:

**21** 22 23 24 **25**  
20  **7**  8  **9** 10  
19  6  **1**  2 11  
18  **5**  4  **3** 12  
**17** 16 15 14 **13**

It can be verified that the sum of the numbers on the diagonals is 101.

What is the sum of the numbers on the diagonals in a 1001 by 1001 spiral formed in the same way?

26. Consider all integer combinations of *ab* for 2 ≤ *a* ≤ 5 and 2 ≤ *b* ≤ 5:

22=4, 23=8, 24=16, 25=32  
32=9, 33=27, 34=81, 35=243  
42=16, 43=64, 44=256, 45=1024  
52=25, 53=125, 54=625, 55=3125

If they are then placed in numerical order, with any repeats removed, we get the following sequence of 15 distinct terms:

4, 8, 9, 16, 25, 27, 32, 64, 81, 125, 243, 256, 625, 1024, 3125

How many distinct terms are in the sequence generated by *ab* for 2 ≤ *a* ≤ 100 and 2 ≤ *b* ≤ 100?

27. Surprisingly there are only three numbers that can be written as the sum of fourth powers of their digits:

1634 = 14 + 64 + 34 + 44  
8208 = 84 + 24 + 04 + 84  
9474 = 94 + 44 + 74 + 44

As 1 = 14 is not a sum it is not included.

The sum of these numbers is 1634 + 8208 + 9474 = 19316.

Find the sum of all the numbers that can be written as the sum of fifth powers of their digits.

28. In England the currency is made up of pound, £, and pence, p, and there are eight coins in general circulation:

1p, 2p, 5p, 10p, 20p, 50p, £1 (100p) and £2 (200p).

It is possible to make £2 in the following way:

1×£1 + 1×50p + 2×20p + 1×5p + 1×2p + 3×1p

How many different ways can £2 be made using any number of coins?

29. We shall say that an *n*-digit number is pandigital if it makes use of all the digits 1 to *n* exactly once; for example, the 5-digit number, 15234, is 1 through 5 pandigital.

The product 7254 is unusual, as the identity, 39 × 186 = 7254, containing multiplicand, multiplier, and product is 1 through 9 pandigital.

Find the sum of all products whose multiplicand/multiplier/product identity can be written as a 1 through 9 pandigital.

HINT: Some products can be obtained in more than one way so be sure to only include it once in your sum.

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30. The fraction 49/98 is a curious fraction, as an inexperienced mathematician in attempting to simplify it may incorrectly believe that 49/98 = 4/8, which is correct, is obtained by cancelling the 9s.

We shall consider fractions like, 30/50 = 3/5, to be trivial examples.

There are exactly four non-trivial examples of this type of fraction, less than one in value, and containing two digits in the numerator and denominator.

If the product of these four fractions is given in its lowest common terms, find the value of the denominator.